Traffic Flow Modeling and Car Accident Risk

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Background

- Traffic modeling uses PDEs to simulate traffic flow
- Used to predict and prevent traffic jams
- Most models assume car crashes cannot happen
- However, predicting accidents is also very important
- Crashes can be permitted using a coupled model
- Combination of a classic traffic model and gas flow model

The Aw-Rascle Model

- Second-order model: velocity (v), density (ρ)
- Anticipation factor (p) analogous to pressure
- First equation conserves mass
- Second equation conserves momentum
- Has a maximum density, no collisions possible

$$\partial_t \rho + \partial_x(\rho v) = 0,$$
$$\partial_t(\rho w) + \partial_x(\rho w v) = 0,$$
$$w = v + p(\rho).$$

The Pressureless Gas Dynamics Model

$$\partial_t \rho + \partial_x(\rho v) = 0,$$
$$\partial_t(\rho v) + \partial_x(\rho v^2) = 0,$$

- Comes from gas flow modeling
- Very similar structure to AR model
- $w = v + p(\rho)$ replaced by v
- Since drivers do not anticipate traffic, crashes can occur
- Delta shocks, where density increases without bound

Numerical Approximation Methods

- McCormack Scheme
- $U = (\rho, \rho w)^T$, $F = vU$

Predictor

$$\tilde{U}_i^{n+1} = U_i^n - \frac{\mathrm{dt}}{\mathrm{dx}}\left(F_i^n - F_{i-1}^n\right) \qquad (7)$$

Corrector

$$U_i^{n+1} = 0.5\left(U_i^n + \tilde{U}_i^{n+1}\right) - 0.5\frac{\mathrm{dt}}{\mathrm{dx}}\left(\tilde{F}_{i+1}^{n+1} - \tilde{F}_i^{n+1}\right) \quad (8)$$

$$\partial_t \rho + \partial_x(\rho v) = 0,$$
$$\partial_t(\rho w) + \partial_x(\rho w v) = 0,$$
$$w = v + p(\rho).$$

- Requires a smoothing step to reduce spurious oscillation

Numerical Approximation Methods

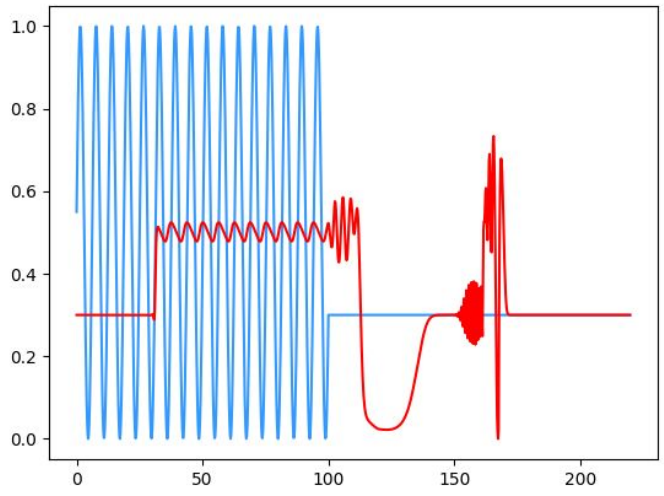
- Godunov-Type Scheme
- $Q = (\rho, \rho v)^T$, $F = vU$
- Uses values at interfaces

$$\partial_t \rho + \partial_x(\rho v) = 0,$$
$$\partial_t(\rho v) + \partial_x(\rho v^2) = 0,$$

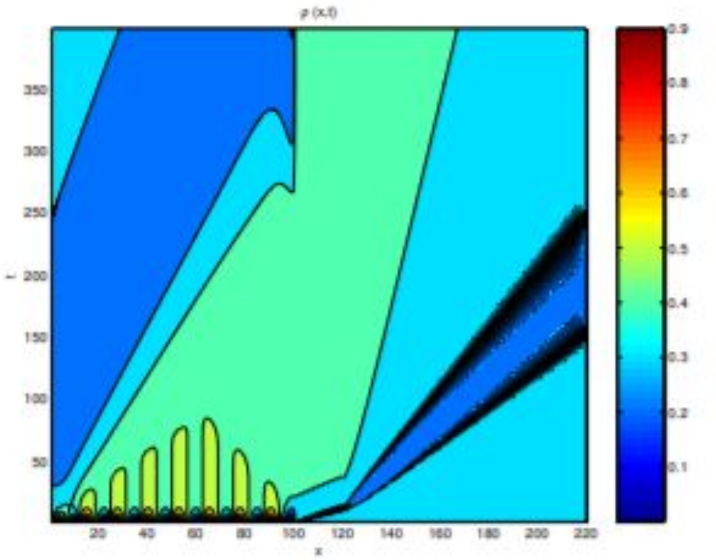
$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}(F_{i+1/2} - F_{i-1/2}) - \frac{\Delta t}{\Delta x}(\tilde{F}_{i+1/2} - \tilde{F}_{i-1/2})$$

- Correction term gives second order accuracy

Numerical Approximation Methods

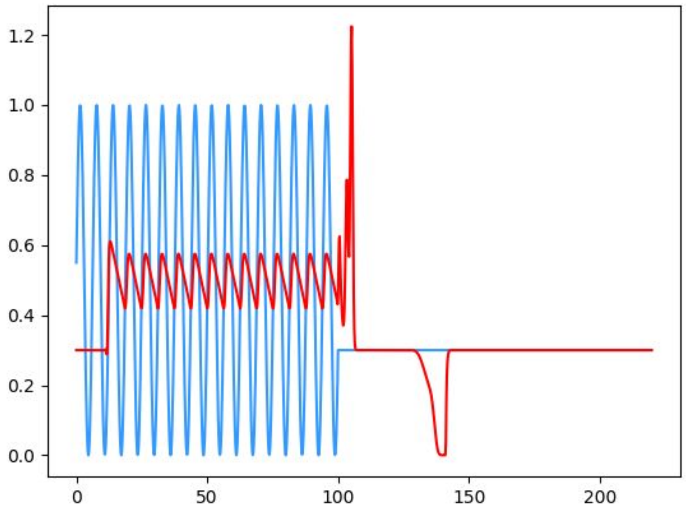


My model, using McCormack and Godunov schemes
Graphs position vs density at t=0 and t=150



A model using Godunov schemes only
Graphs position vs. time, density as color

Numerical Approximation Methods



Same situation as before, but with lower velocity in the PGD zone - an accident occurs
Graphs of position vs density are at t=0 and t=65

References

- Modeling road traffic accidents using macroscopic second-order models of traffic flow
- The Dynamics of Pressureless Dust Clouds and Delta Waves
- Improved Numerical Method for Aw-Rascle Type Continuum Traffic Flow Models