Penn
UNIVERSITY of PENNSYLVANIA

Math 370 in-class exam

October 23, 2017

No books, paper or any electronic device may be used. Please turn off your phones, and put all electronic devices, including calculators and phones, in your bags.

This examination consists of eight questions plus one extra credit problem. Partial credits will be given for *substantial* work.

Please **show all your work**.

NAME (PRINTED):

INSTRUCTOR: C.-L. Chai

My signature below certifies that I have complied with the University of Pennsylvania's *code of academic integrity* in completing this examination.

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Your signature

1	2	3	4	5	6	7	8	9	Total

1. (10 pts) Prove that every subgroup of a cyclic group is itself a cyclic group. (Saying that this is a theorem in the textbook doesn't do.)

2. (10 pts) Give an example of a non-commutative finite p-group and a non-commutative finite q-group for two *distinct* prime numbers p, q.

3. (10 pts) True or false: Every group with 55 elements is commutative. (Give a proof if the statement is true, and give a counter-example if the statement is false.)

4. (10 pts) Give an example of an infinite dimensional vector space V over a field F. (Please give a complete proof that your example is indeed infinite dimensional.)

5. (10 pts) True or false: The complex group ring $\mathbb{C}[G]$ of a finite group G is commutative if and only if G is commutative. (Give a proof if the statement is true, and give a counter-example if the statement is false.)

6. (10 pts) Give an example of a ring R and an ideal I of R such that I is *not* a principal ideal. (You need to show that the examples you give have the required properties.)

7. (20 pts) Find all ideals of the group ring $\mathbb{C}[\mathbb{Z}/2\mathbb{Z}]$.

8. (20 pts) Find explicitly a Sylow 3-subgroup of S_4, and determine the number of all Sylow 3-subgroups of S_4.

9. (20 extra points) Find explicitly a Sylow 2-subgroup of S_4, and determine the number of all Sylow 2-subgroups of S_4